by using ion propulsion after 1975. Considering the fact that the European launching capacity is likely to remain limited for financial reasons, CNES plans to concentrate its effort on 1) hydrazine catalyst decomposition thrusters for satellites to be launched after 1972, and 2) ion engines for satellites to be launched after 1975.[11]

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Stochastic Crew Motion Modeling

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For the design of attitude control systems for large spacecraft, it is desirable to have a statistical description of the disturbing forces and moments resulting from crew motion. Through extensive simulation programs, data have been obtained which are representative of forces and moments that would result from typical crew activities. An analysis has shown the data to be stationary, meaning that equivalent data with the same frequency and rms amplitude characteristics can be generated by passing white noise through linear filters. Filter parameters were obtained by a special technique of least-squares curve fitting of various filter structure equations to the power spectral density curves of the data. The result is a convenient method for continuously generating crew motion disturbance data for input to computer simulations.

Nomenclature

A_i, B_j	= parameters defined by Eq. (15); $i = 1,2,\ldots, n$;
D^*	= constant denominator defined by Eq. (18)
D	= defined by Eq. (16)
E	= error function defined by Eq. (17)
F_x, F_y, F_z	= forces in the specified directions, lb
$H(j\omega)$	= synthesizing filter transfer function
$\bar{M}_{cm}$	= moment due to the center of mass moving from its initial position, $\bar{M}_{cm}$ $(M_{cmx}, M_{cmy}, M_{cmz})$, ft-lb
M_x, M_y, M_z	= moments due to subject's motion ft-lb
m	= mass of the subject
$P(\omega)$	= power spectral density (PSD) defined by Eq. (4)
$\bar{r}_{cm}$	= position vector locating the c.m. position relative to the load cell centroid, $\bar{r}_{cm}(x_{cm}, y_{cm}, z_{cm})$
$R_T(r)$	= truncated autocorrelation function, Eq. (5)
$\bar{W}$	= subject's weight defined as a vector
Δ	= the denominator of Eq. (14)
σ	= standard deviation of a random variable
μ	= mean of a random variable
$\tau_i, \zeta_i, \omega_i$	= free parameters of the synthesizing filter, $i = 1,2$

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Introduction

As early as 1962, it was recognized by Roberson[1] that with the advent of manned spacecraft the effects of the motion of the crew might produce significant spacecraft disturbances. Later investigations (Murrish and Smith,[2] Poli,[3] and others) have verified Roberson's conclusions. One current example is seen in Skylab, for which analysis has shown that the Apollo telescope mount (ATM) must be mechanically isolated from the basic vehicle in order to obtain the desired pointing accuracies for the ATM solar experiments.

Previous studies of crew motion effects have not resulted in a convenient technique of incorporating the disturbances in ground simulation programs. Normally, the crew motion forces and moments would have to be recorded on digital or analog tape and continuously fed into the simulation programs, thereby requiring a large tape library with the resulting tape handling problems. In the search for a more convenient technique, it was recognized that if the crew motion data could be shown to be stationary, it could then be modeled by synthesizing a filter, which when activated by a white noise source would yield a power spectral density (PSD) approximating that of the experimental data. A nonparametric run and trend test[4,5] was made on the crew motion data generated by Murrish and Smith,[2] and the data did show stationarity, validating the stochastic modeling approach. Such a model has the advantage of being easily implemented on either a digital or analog computer with continuous data being easily generated for any length of simulation time. The digital simulation requires nothing more than a random number generator and a set of difference equations, and the analog simulation requires a white noise source and several integrators to simulate the filter.

The construction of a stochastic crew motion model requires three steps: 1) simulation of the activity under consideration, 2) determination of the PSD of the resultant wave forms, and 3) synthesis of the filters by using a least-squares curve fitting program.

Simulation of Crew Motion Disturbances

Following a suggestion by Roberson,[1] the crew motion forces and moments were generated in a laboratory without regard to the size or rotational state of the spacecraft in which the motions might be performed. The results then are valid only for a spacecraft much larger than an astronaut (e.g., Skylab) and with a nominally zero rotational state. Further analysis is recommended to verify these assumptions if it is desired to apply the results to other type spacecraft.

The types of crew activities most applicable to the intended use of the results are those where the crew member is constrained by harnesses or other restraining devices from gross translational movements of his center of mass (c.m.). Disturbances from these activities can then be measured by the use of a relatively simple mockup covering all areas of the spacecraft that the astronaut ordinarily contacts during performance of a task, and the results can then be applied at a single point in the spacecraft. The laboratory equipment for simulation of these types of activities consisted of the activity station mockup, a force measuring system, and a data recording system.

The activity mockup used for simulation of a control console operation (Fig. 1) is configured according to the lunar module ATM control and display console. Switches equivalent to those presently planned to be used in the ATM were rigidly mounted at various locations on the console and were activated during performance of the console activities.

The equipment for obtaining and recording crew motion data comprises 1) a load-cell array, 2) signal-conditioning circuits, 3) a small analog computer, 4) an analog-to-digital converter, and 5) a digital tape recorder. The load-cell array comprised a load-bearing platform or sensing plate supported by six individual load cells arranged in a geometric array as indicated in Fig. 1. The small analog computer resolves the outputs of the individual load cells into voltages that are proportional to the three forces and three moments applied to the load cell array and are resolved in the load cell orthogonal axis system. The forces and moments act at a point defined as the load cell centroid, which is located at the base of the sensing plate and geometrically centered within a triangle formed by the three load cell attach points. Signals from the analog computer are digitized and tape-recorded.

Reduction of Crew Motion Data

Since the laboratory experiment had to be performed in a gravity environment, the recorded results contain "dynamic" components and "static" components. The "dynamic" forces and moments exist in the data only when the subject

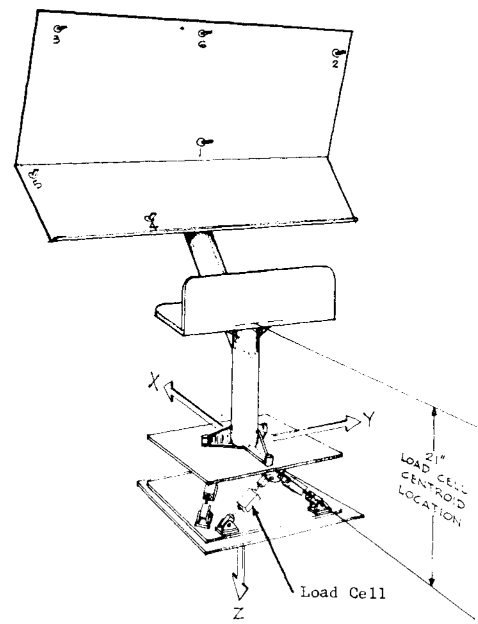

Fig. 1 Sketch of console mockup showing axis and switch location.

has a velocity relative to the load cell array, and they are the only forces and moments that would exist if the experiment were performed in a weightless environment. The "static" components can occur even though the subject is not actually moving. The static forces are merely the weights of the subject and other equipment placed on the load cell

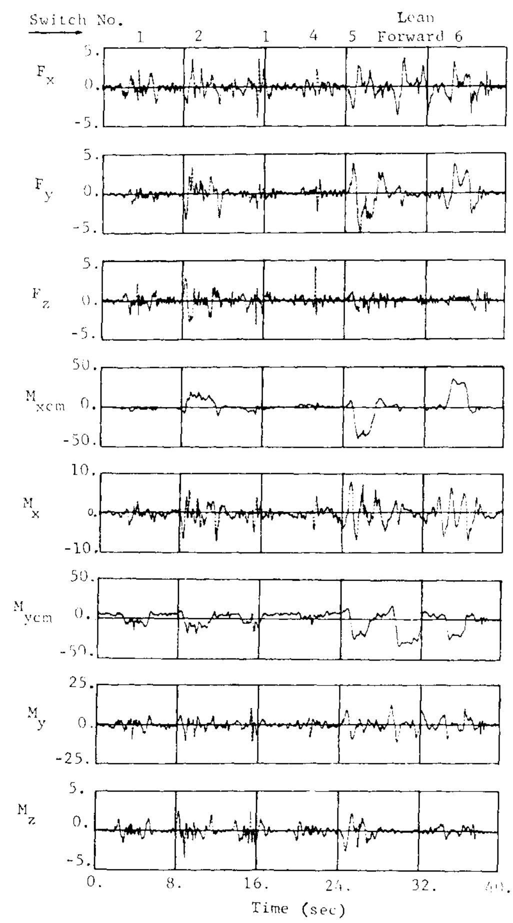

Fig. 2 Force and moment results of console operation activity.

array and are constant, independent of any motion. These forces are easily removed from the data before each run by bias potentiometers, which are part of the signal conditioning circuitry, and as a result these forces do not appear in the data. The static moments, however, vary during a run as a function of the subject's c.m. position. The static moment resulting from his weight W acting through his c.m. at a displacement equal to the c.m. displacement from an initial zero position is given by

$$\bar{M}_{cm} = \bar{r}_{cm} \times \bar{W} \tag{1}$$

and $\bar{r}_{cm}$ can be obtained directly from the load cell forces (x and y components) by application of Newton's law. Since the load cells measure the external forces applied to the array by the subject, the negative equivalents of these forces are exactly those which produce an inertial movement of the subject's combined c.m. The static moments can now be written as

$$Wy_{cm} = M_{xcm} = -g\iint F_y dt dt \tag{2}$$

$$Wx_{cm} = M_{ycm} = +g\iint F_x dt dt,\ M_{zcm} = 0 \tag{3}$$

where g is the acceleration of gravity at the Earth's surface. The dynamic moments are then obtained by subtracting Eqs. (2) and (3), the static moments, from the total moments read by the load cells.

Results for a typical console operation activity are shown in Fig. 2. The resolved forces and moments from the load cell array are plotted along with the dynamic moments M_x and M_y after the static moments have been removed. Several interesting features of these curves should be mentioned. Shown in Fig. 1 are numbered locations of the approximate positions of six switches mounted on the console. At the top of Fig. 2 are numbers that correspond to the switch locations for which the subject reached during performance of this console activity. The two moments M_{xcm} and M_{ycm}, which include c.m. moments, give a clear indication of the subject's motion. For example, between 8 and 12 sec the subject reached for switch 2 (leaning forward and to the right) resulting in a negative M_{ycm} and a positive M_{xcm}. All motions, of course, result in c.m. displacements along the positive x axis, which give consistently negative M_{ycm}s. When the motion includes significant leaning to the right or left, a positive or negative M_{xcm}, respectively, is also obtained.

Power Spectral Density Determination

The power spectrum $P(\omega)$ of a function of time $f(t)$ is defined as the Fourier transform of the autocorrelation function $R(t)$ of $f(t)$; i.e., $P(\omega)$ and $R(t)$ are transform pairs. This definition, relating the autocorrelation and spectral density functions as transform pairs, while correct, gives little insight into the physical interpretation of the PSD of a time series. An intuitively more pleasing definition relates the spectrum to power dissipation per Hertz. If the waveform to be analyzed $f(t)$ is thought of as a voltage, then the PSD of $f(t)$ is the power dissipated in a 1-ohm resistor per unit frequency (v^2/Hz).

There are at least two different approaches in determining the PSD of $f(t)$. One proceeds formally by first taking the autocorrelation of the time series and then the Fourier transform of the autocorrelation function giving the PSD. The other approach can be succinctly described by the equation

$$P(\omega) = \lim_{T\to\infty} \frac{1}{2T} \left| \int_{-T}^{T} f(t) e^{-j\omega t}\, dt \right|^2 \tag{4}$$

and noting that the integral is no more than the expression for the Fourier transform of $f(t)$. Even though this approach seemingly gives the most straightforward definition of the PSD, until recently the Fourier transform approach was not computationally efficient. Previously, the most efficient computational technique was considered to be that obtained via the mean lagged product method of Blackman and Tukey.[6] Basically, this algorithm involves the use of an estimate of the truncated autocorrelation function. This approximation is expressed in terms of the sampled time series $f_T(0), f_T(1), f_T(2), \ldots, f_T(N-1)$ as,

$$R_T(r) = \frac{1}{(N-1)-r} \sum_{q=0}^{N-1-r} f_T(q) f(q+r) \tag{5}$$

where ($r = 0, 1, \ldots m$ where $m \leq N-1$). Using this expression the samples of the approximate autocorrelation function $R_T(r)$ are computed. Since the autocorrelation function is an even function of time, the Fourier transform of $R_T(r)$ is the PSD and can be written as

$$P(n) = \Delta t \left[R_T(0) + 2 \sum_{q=1}^{m-1} R_T(q) \cos\left(\frac{qn}{m}\right) + R_T(m) \cos n \right] \tag{6}$$

Since $P(n)$ is symmetric in n for every integral multiple of m it is only necessary to compute $P(n)$ for $n = 0, 1, \ldots, m$, and the frequency corresponding to n is $n/(2m\Delta t)$. This is sometimes called the Blackman-Tukey method and has been widely used with great success.

The other approach to PSD computation called the Cooley-Tukey method is concerned with obtaining the Fourier coefficients of the original sampled time function by using the fast Fourier transform (FFT). The FFT is a clever computational technique which obtains the coefficients of the discrete Fourier transform (DFT) in a computationally efficient manner. Conventional techniques yield the DFT coefficients in N^2 arithmetic operations, whereas the FFT takes about $2N \log_2 N$, and this number is so small when N (the number of data points) is large as to completely change the computational aspects of obtaining the PSD by Fourier techniques. Using the Cooley-Tukey method as modified by Welch,[7] PSDs were obtained from the experimental crew motion data traces in less than half the computational time required using the Blackman-Tukey mean lag product approach. Also, the computed spectra using both methods are nearly indistinguishable. These results were obtained for a time series containing 3600 points. Typical results of the crew motion spectral estimates are shown in the sequel.

Filter Synthesis

A random process $v(t)$ possessing a constant PSD is referred to as a white noise process, i.e., a process or time function, which is composed of frequency components in equal amounts. White noise sources contained in analog computers are only an approximation to the aforementioned idealization and produce a time function the spectrum of which is essentially flat to some high frequency. The amplitude distribution in Gaussian with negligible mean.

In measuring the characteristics of a noise voltage, $v(t)$ there are certain basic relationships that are very useful. One of these is

$$\bar{v}^2 = \sigma^2 + \mu^2 \tag{7}$$

where $\bar{v}^2$ is the mean square value, σ is the standard deviation, and μ is the mean. If $\mu = 0$ then Eq. (7) degenerates to an equation which states that the rms value and the standard deviation are the same when the mean is zero. For a noise with a zero mean value it can be shown that

$$\sigma^2 = \int_0^\infty P_v(f) df \tag{8}$$

where $P_v(f)$ is the spectral density of the noise process $v(t)$.

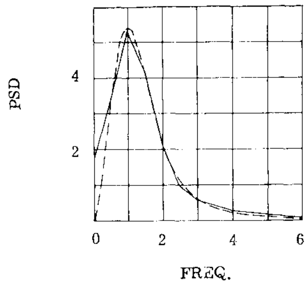

Fig. 3 PSD plot and approximation.

Equation (8) states that the variance of a zero mean stochastic process is equal to the area under the spectral density curve.

Examining the spectral density of the output $w(t)$ of a linear filter with transfer function $H(j\omega)$ when the input $v(t)$ is white noise reveals an interesting relationship between the spectral densities of $v(t)$ and $w(t)$. The power spectrum $P_w(\omega)$ is given by

$$P_w(\omega) = P_v(\omega)|H(j\omega)|^2 \quad (9)$$

When the white noise source has unity spectral density, Eq. (9) becomes

$$P_w(\omega) = |H(j\omega)|^2 = H(j\omega)H(-j\omega) \quad (10)$$

and the mean square value of the output is given by

$$\bar{w}^2 = \int_0^\infty H(j\omega)H(-j\omega)d\omega \quad (11)$$

It is seen, because of the structure of Eq. (10), that $P_w(\omega)$ is an even function of omega; also the rms value of $w(t)$ is obtained as the area under the PSD curve.

Choice of Filter Structure

The filter structure must be sufficiently complex to reproduce the salient features of the PSD being approximated and yet not so complex as to preclude economical computer usage. Since theoretically the spectral density of the class of activities under consideration must be zero at zero frequency, any reasonable filter structure must contain a zero at the origin. Further, unimodal spectral densities are well approximated by a single quadratic in the denominator. The transfer function of such a filter is

$$H_2(s) = \tau_1 s/(s^2 + 2\zeta_1\omega_1 s + \omega_1^2) \quad (12)$$

The corresponding power spectral density when $H_2(s)$ is driven by white noise becomes

$$P_2(\omega) = H_2(j\omega)H_2(-j\omega) = \tau_1^2\omega^2/(\omega^4 + \xi_1^2 + \omega_1^4) \quad (12a)$$

where $\xi_1 \equiv 4\zeta_1^2\omega_1^2 - 2\omega_1^2$. To approximate a power spectral density curve containing two peaks requires a fourth-order polynomial in the denominator, and the transfer function of the corresponding filter can be written as

$$H_4(s) = \tau_1 s/[(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)(s^2 + 2\zeta_2\omega_2 s + \omega_2^2)] \quad (13)$$

and the power spectral density becomes

$$P_4(\omega) = H_4(j\omega)H_4(-j\omega) = \tau_1^2\omega^2/\Delta \quad (14)$$

Table 1 Comparison of filter parameters

Activity	Tau	Omega	Freq	Zeta
M_z (Console operation)	3.96	5.9	0.94	0.622
M_x (Console operation)	2103.3	4.36	0.69	0.279
		18.5	2.94	0.328

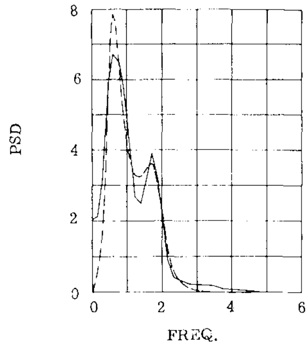

Fig. 4 PSD plot and approximation.

where

$$\Delta = \omega^8 + (\xi_1 + \xi_2)\omega^6 + (\xi_1\xi_2 + \omega_1^4 + \omega_2^4)\omega^4 + (\xi_1\omega_2^4 + \xi_2\omega_1^4)\omega^2 + \omega_1^4\omega_2^4$$

where $\xi_2 \equiv 4\zeta_2^2\omega_2^2 - 2\omega_2^2$. It has been shown that the general form assumed by the spectral density curve is represented by the equation

$$P(\omega^2) = \frac{A_1\omega^2 + A_2\omega^4 + \ldots + A_n\omega^{2n}}{1 + B_1\omega^2 + B_2\omega^4 + \ldots + B_m\omega^{2m}} \quad (15)$$

where the A's and B's are functions of the filter parameters.

It is instructive at this time to review the steps required to synthesize the crew motion filter. First, experimentally generated crew motion data traces were obtained from the restrained crew motion simulator. The effects of gravity were numerically removed from the force and moment data and these sampled time functions (time series) were transformed into the frequency domain via the FFT to obtain their PSDs. Depending upon the shape of the PSD curves (unimodal or bimodal) a particular filter structure was chosen; either the second- or fourth-order form was judged adequate. Then the resultant spectral density curve was adjusted by changing the filter parameters to obtain a least-squares fit with the PSDs obtained from the experimental data traces.

PSD Curve Fit by Least-Squares

To curve-fit the PSD data with an equation of the form given by Eq. (15) using least-squares techniques required defining a special weighted error function. The usual error function, given by $e = P_c - P$ where P_c represents the PSD data, results in least-squares equations that cannot be explicitly solved for the constants A_j and B_j. Following Gagné[8] the weighted error function eD was tried, where D is the denominator of Eq. (15), i.e.,

$$D = 1 + \sum_{j=1}^{M} B_j\omega^{2j} \quad (16)$$

This results in least-squares equations which are readily solvable; however, a poor fit was usually obtained since data over all frequencies were not weighted equally. Therefore, a second error function was defined as

$$E = eD/D^* \quad (17)$$

where

$$D^* = 1 + \sum_{j=1}^{M} C_j\omega^{2j} \quad (18)$$

The constants C_j in D^* are assumed to be known and thus adding D^* does not add to the complexity of the least-squares equations. Then, if the C_j's can be made equal to the B_j's, an exact least-squares fit by the usual definition of the error can be obtained. This was accomplished in the least-squares algorithm by an iterative scheme which continually replaces the C_j's by the newly calculated B_j's until the constants approach each other to within a specified error.

Comparisons and Conclusions

Using the theory previously developed, filters were synthesized for the forces and moments of ten different activities. Typical results are shown in Figs. 3 and 4. Figure 3 shows the spectral density of the moment about the z axis for console operation (solid line) and the corresponding filter approximation (dotted line). Since the PSD being approximated contains only one peak, a second-order filter form was used. Figure 4 shows the PSD of the moment about the x axis and the corresponding fourth-order filter approximation. The filter parameters corresponding to Figs. 3 and 4 are shown in Table 1, where it is seen that the PSD's peak value occurs very close to ω_n and the bandwidth of the peak varies directly with ζ_n. Figures 3 and 4 demonstrate (in fact good approximations were obtained for all forces and moments corresponding to the ten restrained activities investigated) that simple stochastic models can be used. It should be noted, however, that the approximation is performed in the frequency domain, and the resultant time function does not necessarily correspond to the time function being approximated.

In conclusion, good approximations to the PSDs for all crew motions considered were obtained using a linear filter driven by white noise. The least-squares approach to selecting the filter parameters yielded good results in every case. It is to be noted that the least-squares approach to spectrum shaping is not restricted to crew motion modeling, but is amenable to any application which requires a signal with specified spectrum.

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